MODELS FOR MESSAGE TRANSMISSIONS AND INTERFERENCE-CAUSED RETRANSMISSION THROUGH A MULTI-CHANNEL SATELLITE COMMUNICATIONS SYSTEM

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THESIS

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Multi-Channel Satellite Communications System

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ABSTRACT

A satellite communication system has been proposed for use in improving naval communications. This system would feature random selection of channels by originators. For a message to be received successfully it must be alone on a channel during its transmission period. The number of transmissions by each unit and the total system transmission time must be minimized to prevent detection by enemy direction finding equipment. An expression is developed for the probability that a specified number of messages, s, of n original messages, will be received successfully after being transmitted a number of times, m. An expression is derived for the probability that a specified number of messages, j, of n original messages, will be received successfully on the first transmission by some arbitrary time, t. A Monte Carlo computer simulation is conducted and is the basis for expressions for the maximum number of times any unit would have to transmit to be received successfully.

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I. INTRODUCTION

Improvements in naval communications have been a continuous source of study for many years. Since the time that the importance of communication between one naval unit and another was realized, many important improvements have been made, one of the most significant of these being creation of communications control centers to regulate and relay the tremendous flow of traffic necessary to maintain an effective naval force.

Although the communications control station is a definite improvement over previous systems of control, there still remains considerable delays in this system due to priorities, requests to transmit, acknowledgements, etc. What is needed is a system to receive a large volume of traffic in a short amount of time with the least amount of delay for the transmitting units.

It has been proposed that a multi-channel satellite communication system be implemented for this purpose. The number of satellites needed would be sufficient to cover all areas where naval units might be located. Each satellite would contain several frequency channels designated as service channels and one or two channels as control channels. Messages to be transmitted to other stations would be sent over the service channels for receipt or relay by the control station and the control channels would be used for such functions as message acknowledgement, retransmission orders, and channel assignments.

The original application of this type of system is classified and is described by Crigler [1]. However, this system may lend itself to the ship-to-shore communications problem. With the advent of electronic warfare, minimizing transmissions has become critical in keeping unit location secure from the enemy. At the same time there is still a certain amount of traffic which must be sent to maintain accountability and control.

To be effective, a communications system must require a minimum amount of transmission time from each unit, hence keeping total system time low. This would make detection of a transmitting unit by enemy forces more difficult and improve the unit's combat effectiveness. At the same time, however, system operations must have a high probability of success. It would accomplish little to have a system which requires a small amount of transmission time if the probability of success of the transmitted messages is not high.

This thesis is a study of one of the possible methods a satellite communications system could be used to keep unit and system transmission time low and keep the probability of system success high.

II. SYSTEM OPERATION

The system to be studied would feature random selection of a service channel from all those provided. The control channel serves only to relay message acknowledgements or to indicate that message retransmission is necessary. When an originator desires to send a message over the system, he randomly selects a service channel, tunes his transmitter to the frequency for that channel, and transmits his message.

All messages originated would be automatically encrypted during transmission on the service channel. Automatic check-decryption equipment at the control center would rapidly process the messages upon receipt. If the message decrypts without error (garble), this equipment would send back a coded response to the originator via the control channel. If a message does not decrypt properly, a different coded response would be sent and the originator would have to transmit the message again.

If the coded response is positive, the message has been received ungarbled and the originator is finished. If he receives a negative acknowledgement, he must reselect a channel, retune his transmitter, and retransmit. This procedure continues until he receives a positive acknowledgement or he has transmitted the maximum number of times allowed or has used the maximum time allowed a unit by a preset policy for this system.

A negative acknolwedgement is received if two or more messages are being transmitted on the same channel at the same time. If this occurs, all messages involved will be garbled and must be retransmitted. Each unit involved will receive a negative acknowledgement and must reselect a new channel at random, independent of each other, and transmit again.

Estimated time requirements for operations in the system are presented in the following table. Times indicated are for a single unit and are in seconds.

FUNCTION		TIME REQUIRED
Select Channel and Tune Transmitter		30 sec.
Transmit Message		15 sec.
Receive Acknowledgement		2 sec.
	Total	47 sec.

III. STATEMENT OF PROBLEM

It is desired to send one message each from n different originators over a c-channel satellite communications system using random selection of service channels. It is also desired that the probability that each message is received ungarbled on the first attempt is high. If a message is not received successfully on the first attempt, it will be retransmitted later on another randomly selected channel.

For the system to be successful all messages must eventually be received ungarbled. This must be accomplished within a reasonable time after the originators begin transmitting to keep system broadcasting time low and minimize the possibility of electronic detection by the enemy. Similarly, the number of attempts by each unit must be minimized.

This thesis is directed toward providing models which can be used in the design of the system. Section V is devoted to the mathematical development of the probability of successfully receiving any number of the n-messages on the first transmission with an arbitrary time, t. Another formula is derived to determine the probability of receiving any number of a particular generation of messages. A generation of messages is the set of all transmissions that had to be repeated the same number of times. Thus, the zeroth generation is the set of the n original messages, the first generation is the set of all first retransmissions, and so on. Section VI B is a development based on a Monte Carlo computer simulation of the maximum number of times a

unit would have to transmit to have its message received successfully on a c-channel n-original system.

IV. ASSUMPTIONS

The following assumptions were made in the development of the analytical and computer simulation models:

(1) All messages are considered to be of equal length, d. This is a rather restrictive assumption and is used to simplify the development of the models. The assumption was originally made to apply to coded format responses where all messages were designed to be of equal length. It was shown by Kabak [2] in a problem mathematically similar to this one that the probability of non-interference does not change significantly when the shape parameter of the Erlang distribution used for service time is increased to infinity, i.e., even when service times (message lengths) are constant. Kabak's model, however, assumes one channel only.

(2) Frequency channel selection by a user is considered to be completely random. Random selection methods are increasingly in use by the Navy and although it is rare that one can select an alternative completely at random the assumption is made for this development. In this case all service channels are equally likely to be selected.

(3) Interference occurs if the transmission start times of two messages on the same channel are within d seconds of each other. If the two times are d seconds or more apart, interference does not occur. If interference does occur, both messages must be retransmitted on a new randomly selected channel after a delay to reselect a new random channel and

retune the transmitter. For numerical computations, $d = 15$ secs will be used.

(4) Transmitter selection and tuning times are constant for all originators. For numerical computations, $h = 47$ secs will be used.

(5) Beginning times of transmissions are uniformly distributed over an interval $(0,L)$. All originators are considered to be of comparable readiness and training status. The interval is considered to be of reasonably short duration. See [1] for detailed reasons. For numerical computations, $L = 300$ secs will be used.

V. ANALYTICAL RESULTS

Define,

A_k = the number of messages attempted in the k^{th} generation, $k = 0,1,\ldots$

N_k = the number of messages successfully received in the k^{th} generation, $k = 0,1,2,\ldots$

The probability that any specified number, j, of the n messages transmitted over a c-channel system will be received on the first transmission is developed by Crigler [Ref. 1] and is given by

$$P[N_0=j] = \sum_{m_i} \sum_{b_i} \binom{n}{m_1 m_2 \ldots m_c} \left(\frac{1}{c}\right)^n \prod_{i=1}^{c} P[B_i=b_i|M_i=m_i] \tag{1}$$

where

N_0 = the number of messages received successfully on the first transmission (zeroth generation) and the two sets of multiple sums are over all m_i and b_i, $i = 1,2,\ldots,c$, such that $\sum_i m_i = n$ and $\sum_i b_i = j$. $P[B_i=b_i|M_i=m_i]$ is given as

$$= \sum_{q=b_i}^{\left[\frac{m_i+b_i-2}{2}\right]} \binom{m_i-q-2}{q-b_i} \binom{q+1}{b_i} \sum_{s=q}^{\left[\frac{L}{d}\right]} (-1)^{s-q} \binom{m_i-q-1}{m_i-s-1} \left(1-\frac{sd}{L}\right)^{m_i} ;$$

$$0 \le b_i \le m_i - 2 \tag{2}$$

$$= 0 \qquad ;\ b_i = m_i - 1$$

$$= \left(1 - \frac{(m_i - 1)d}{L}\right)^{m_i} \quad ; b_i = m_i$$

where,

B_i = the number of messages received successfully on channel i.

M_i = the number of messages attempted on channel i.

d = message length.

L = length of transmission interval over which transmission start times are uniformly distributed.

This development was made under the assumption that messages on one transmission generation do not interfere with messages on any other transmission generation. Original messages (zeroth generation) are received over the interval (d,L+d). First retransmissions (first generation) are received over the interval (d+h, L+d+h), an interval equal in length, L, to the zeroth generation interval but shifted to the right by an amount of time, h, the time it takes to receive an acknowledgement (2 sec), randomly select a new channel and retune the transmitter (30 sec), and retransmit the message (15 sec). Similarly, each succeeding generation is received over an interval of length, L, shifted to the right by an amount, h, from the interval of the previous generation.

The number of attempted message transmissions in the first generation is dependent only on the results of the zeroth generation and is equal to A_0-N_0. The transmission start times of the first generation are uniformly distributed

over the interval (h, L+h). Similarly, the m^{th} generation is dependent only on the results of the (m-1)st and the transmission start times are uniformly distributed over the interval (mh, L+mh).

Now, since

$$A_0 = n$$

then

$$A_1 = A_0 - N_0 = n - N_0$$

$$A_2 = A_1 - N_1 = n - (N_0+N_1)$$

$$A_3 = A_2 - N_2 = n - (N_0+N_1+N_2)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$A_m = A_{m-1} - N_{m-1} = n - \sum_{i=0}^{m-1} N_i .$$

Using the assumption that different generation messages do not interfere with each other, the distribution of the number of attempted messages in any generation may be derived.

Now, for m = 1,2,...

$$\begin{aligned} P[A_m=r] &= \sum_{\ell=r}^{n} P[A_m=r|A_{m-1}=\ell]P[A_{m-1}=\ell] \\ &\sum_{\ell=r}^{n} P[A_{m-1}-N_{m-1}=r|A_{m-1}=\ell]P[A_{m-1}=\ell] \\ &\sum_{\ell=r}^{n} P[N_{m-1}=\ell-r|A_{m-1}=\ell]P[A_{m-1}=\ell] \quad r = 0,1,\ldots,n. \end{aligned} \tag{3}$$

The conditional probability $P[N_{m-1}=\ell-r \mid A_{m-1}=\ell]$ can be found using Equation (1) with j replaced by $\ell-r$ and n replaced by ℓ. Since the distribution of A_0 is the degenerate distribution at the value n, those of $A_1, A_2, \ldots$ may be successively determined from Equation (3) with the repeated use of Equation (1).

The probability that a specified number of messages, s, is received successfully on any generation, m, is given by

$$P[N_m=s] = \sum_{r=s}^{n} P[N_m=s \mid A_m=r]P[A_m=r] \quad s= 0,1,2,\ldots,n \tag{4}$$

where the conditional probability $P[N_m=s \mid A_m=r]$ can be found again from Equation (1) with j replaced by s and n replaced by r. $P[A_m=r]$ is found by using Equation (3) with $r=s$.

The probability that a specified number of messages, j, are successfully received on the first transmission by some arbitrary time, t, can be found by modifying Equation (1). The transmission receipt times are uniformly distributed for the zeroth generation over the interval (d, L+d) and the number of messages attempted is n.

The probability that the number of messages attempted by time $t \le L+d$ is some value, k, is given by

$$P[A_0(t) = k] = \binom{n}{k}\left(\frac{t}{L+d}\right)^k\left(1-\frac{t}{L+d}\right)^{n-k} \qquad \begin{array}{l} t \le L+d \\ k = 0,1,2,\ldots,n. \end{array}$$

Finally, the probability that a specified number of messages, j, are received successfully on the first transmission by some time, t, is given by

$$P[N_0(t) = j] = \sum_{k=0}^{n} P[N_0(t) = j | A_0(t) = k] P[A_0(t) = k] \quad (5)$$

$$t \leq L+d$$

$$k = 0,1,2,\ldots,n.$$

Here the conditional probability $P[N_0(t) = j|A_0(t) = k]$ is given by Equation (1) after replacing L by t-d and n by k.

For time greater than L+d, the probability that any specified number of messages will be received successfully on the first transmission is the same as for $t = L+d$ since all n of the original transmissions have been received by $t = L+d$.

An example of the results of Equation (5) is given in the following table:

TABLE I. $P[N_0(t) = j]$
$n = 2$, $c = 2$, $d = 15$ sec., $L = 300$ sec.

j \ t	50	100	150	200	250	300
0	.697	.465	.274	.143	.072	.049
1	.281	.445	.500	.445	.281	.000
2	.010	.094	.226	.417	.648	.951

VI. COMPUTER SIMULATION

A. SIMULATION

1. Components

The computer simulation is done on an IBM 360 computer using FORTRAN IV. The program utilizes two random number generators, a storage vector (X), a transmission matrix (XT), and a comparison matrix (DIFF). No special assumptions are made about interference between generations of transmission, i.e., messages of different generations are allowed to interfere with each other.

The storage vector (X) is a 49 position vector used to store all transmission start times in the order of their transmission. The transmission matrix (XT), is a cx49 matrix used to indicate the channel on which a message is transmitted. The comparison matrix (DIFF) is a cx49 matrix used to compare transmission start times of messages on the same channel of the transmission matrix to see if they interfere.

2. Operation

N uniform random numbers over (0,L) are generated and rearranged in increasing order. These numbers represent the transmission start times for the original n messages and are placed in the first n positions of the storage vector (X). The first start time in the vector is assigned one of the c-channels using the second random number generator and is placed in the transmission matrix (XT) in the

first position of the row corresponding to the channel assigned. The other (c-1) rows are assigned a zero in the first position. The second start time in the storage vector is randomly assigned a channel and placed in the transmission matrix in the second position of the row corresponding to its channel and the other (c-1) rows are assigned a zero in the second position. If the two transmission start times are not assigned to the same channel there is no chance for interference and the third start time is placed in the transmission matrix in the third position of the row corresponding to its channel with zeros in the third position of the other (c-1) rows.

If the first two start times are assigned to the same channel, there is a chance for interference and the times must be checked. The comparison matrix (DIFF) compares the two start times to see if they are less than D seconds apart. If the two times are at least D seconds apart, interference does not occur and the third start time in the storage vector is placed in the transmission matrix. If the two times are less than D seconds apart, they are both garbled and must be retransmitted. The delay time to start a retransmission (H) is added to the start time of both of the garbled messages. This additional time (H) is the time to complete the transmission (15 sec), receive a reply from the control center (2 sec), and reselect a channel and retune the transmitter (30 sec). The new retransmission start time for the first message is then fitted into the storage vector

(X) for later transmission. Fitting the retransmission start time into the storage vector requires checking the vector to find the first start time greater than the retransmission start time to be inserted. All start times greater than the one to be inserted are then shifted one position higher in the vector and the retransmission start time is placed in the vacant position. This allows all messages to be retransmitted in the proper order. The retransmission start time of the second garbled message is placed in the storage vector in the same manner. If a retransmission start time is greater than any start time in the vector, it is placed in the next position after the highest start time in the vector.

Once the retransmission start times have been inserted into the storage vector (X) for later transmission, the third transmission start time is placed in the transmission matrix in the row corresponding to the channel selected. The row is then checked back from the third position to see if there are any other transmission start times on the channel. If there is one or more start times on the channel, the most recent is compared with the time in third position as described above and the messages are retransmitted if necessary. Note that it is necessary to check only back from the third position since all later positions in the transmission matrix are still empty. Also, it is necessary to compare only the most recent start time since any other start time which might interfere with the time in the

third position will have interfered with the more recent time also and has been retransmitted already. If there are no other transmission start times on the channel or if interference does not occur, the next start time in the storage vector is placed in the transmission matrix.

The above procedure is repeated until all transmission start times are placed in the transmission matrix and no more interference occurs. When this is completed, all n original messages have been successfully transmitted. Note that in inserting retransmission start times, if a message interfers and its retransmission start time is inserted into the storage vector, should this message interfere again later, its retransmission start time is already in the storage vector and is not reinserted. Several combinations of channels-originals were simulated with each combination iterated 50 times (see Table II).

The computer program and a sample of the printout results showing the storage vector (X) retransmission start times, the storage vector (X) with all transmission start times at the end of the process and the transmission matrix (XT) showing all transmission start times in their appropriate channel positions for the 10 channel - 20 original case are located on pages 28-33.

B. SIMULATION RESULTS

The following definitions are used in evaluating the simulation results:

$\hat{N}(t)$ - the average number of messages received ungarbled in the interval (0,t) in 50 iterations.

$\hat{A}(t)$ - the average number of messages received (garbled and ungarbled) in the interval (0,t) in 50 iterations.

L' = L+d - The time it takes to receive the n original messages.

h - the time required to receive acknowledgement from the control station, reselect a channel, retune the transmitter, and retransmit the message.

M - the number of generations of transmissions needed for all messages to be received successfully. M is a random variable with possible values 0, 1, 2, ...

The average number of completed transmission attempts, $\hat{A}(t)$, in an interval (0,t) is determined by adding the total number of transmissions in the fifty iterations over the interval and dividing by fifty. The average number of successful transmissions, $\hat{N}(t)$ is determined in a similar manner.

Define,

$$K(t) = \frac{\hat{N}(t)}{\hat{A}(t)} .$$

The simulation printouts show that K(t) approaches a constant, K, and this limit is reached for t = 2L'/3 in each of the channel-original combinations. Table II shows the values of K(t) for all c-n combinations simulated for increasing time intervals and the value that K(t) approaches.

TABLE II. d = 15 sec, H = 47.0 sec, L' = 315 sec

$$K(t) = \frac{\hat{N}(t)}{\hat{A}(t)}$$

c-n \ t	50	100	150	200	250	300	350	$K = \lim_{t\to\infty} K(t)$
5-20	.83	.73	.67	.61	.55	.52	.51	.51
7-20	.83	.80	.78	.75	.73	.72	.72	.72
10-20	.90	.86	.82	.81	.80	.79	.79	.80
10-15	.94	.86	.83	.83	.85	.85	.85	.85
10-10	.99	.98	.96	.94	.93	.94	.94	.94
10-5	.99	.91	.86	.90	.89	.91	.91	.91
5-10	.92	.87	.83	.82	.84	.83	.83	.83

Define,

t' - the time it takes for all messages to be received successfully. Then clearly $t' \geq L'$.

K - transmission constant = $\lim_{t\to\infty} K(t)$.

Then,

$$\hat{N}(t') = \hat{A}_0(t') = n. \qquad (6)$$

Since $t' \geq 2/3\ L'$,

$$K \simeq K(t') = \frac{\hat{N}(t')}{\hat{A}(t')} = \frac{n}{\hat{A}(t')}$$

and

$$\hat{A}(t') = \left[\frac{n}{K}\right] \qquad (7)$$

may be used as an approximation, where brackets indicate the largest integer value less than or equal to (n/K).

Now,

$$\hat{A}(t') = \hat{A}_0(t') + \sum_{i=1}^{M} \hat{A}_i(t')$$

or the average number of transmissions in the interval $(0,t')$ is the sum of the average number of original transmission in $(0,t')$ and the average number of retransmissions in $(0,t')$.

From Equations (6) and (7), then

$$\sum_{i=1}^{M} \hat{A}_i(t') = \left[\frac{n}{K}\right] - n = \left[\frac{n}{K} - n\right] .$$

Let

$$\hat{S}(t') = \sum_{i=1}^{M} \hat{A}_i(t')$$

and

$\hat{M}$ = maximum number of generations needed to transmit all messages successfully.

Some reflection shows that

$$\hat{M} = \left[\frac{\hat{S}(t')}{2}\right] ,$$

or the maximum number of generations to complete some number, j, of retransmissions is the largest integer value less than or equal to $(j/2)$. This is found by examining the possible combinations of generations which can be used for any number, j, of retransmissions and taking the largest number of generations needed as the value, $\hat{M}$.

Thus,

$$\hat{M} = \left[\frac{\hat{S}(t')}{2}\right] = \left[\frac{\frac{n}{K} - n}{2}\right] = \left[\frac{n(1-K)}{2K}\right] .$$

Define t^* = time to complete $\hat{M}$ generations.

$$t^* = L' + \hat{M}h$$

or

$$t^* = L' + \left[\frac{n(1-K)}{2K}\right]h$$

and t^* is the maximum amount of time it would take to complete the generations required for all messages to be received successfully. Numerical values for K, $\hat{A}(t')$, $\hat{S}(t')$, $\hat{M}$, and t^*, derived using the simulation outputs are shown in Table III.

TABLE III. d = 15 sec, L' = 315 sec, h = 47.0 sec

t / c-n	K	$\hat{A}(t')$	$\hat{S}(t')$	$\hat{M}$	t*
5-20	.51	39	19	9	738.0
7-20	.72	27	7	3	456.0
10-20	.80	25	5	2	411.0
10-15	.85	17	2	1	362.0
10-10	.94	11	1	0	315.0
10-5	.91	6	1	0	315.0
5-10	.83	12	2	1	362.0

VII. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

A. CONCLUSIONS

Based on the preceeding sections the following conclusions are made:

(1) The probability that any specified number of messages, s, will be successfully received on a subsequent retransmission is given by Equation (4) of Section V. This expression can be used in the system design to determine the probability of success of messages transmitted more than once. It can be used also to aid in deciding the maximum number of transmissions to allow each unit.

(2) If there are n message originators who must each transmit a message of length, d, over a c-channel satellite system, and if all n messages are to begin transmissions by some time, L, subject to the assumptions in Section IV, then the probability that a specified number, j, of the n messages will be successfully received on the first transmission any time, $t \leq L+d$ is given by Equation (5) of Section V.

(3) The ratio of the average number of messages received ungarbled over an interval (0,t) and the average number of messages sent over the same interval approaches a constant at some time greater than or equal to two-thirds the time it takes the n originals to be received. This constant is called the transmission constant. This ratio takes into account all possible methods of interference and all generations of transmissions. This implies that the system appears to reach steady state at some time $t = 2/3\ L'$.

(4) The number of retransmissions for the system to be successful can be determined by using the number of originators and the transmission constant. This value is useful in the system design to determine the proper number of channels to avoid excessive retransmissions.

(5) The maximum number of transmission generations needed for all original messages to eventually be received ungarbled can be determined by using the transmission constant and the number of originators. This value is important in determining the maximum number of times each unit may have to transmit to complete all original messages. From this value, such tactics as emission control plans could be developed.

(6) The maximum time for generations needed for successful completion is determined by using the original message receipt interval, number of originals, transmission constant and time to retransmit. This value can be used to predict the maximum amount of time the system would be exposed to enemy detection under different channel-originator combinations. It could also be used to develop scheduling plans for the satellite's use by groups of originators.

B. RECOMMENDED TOPICS FOR FURTHER STUDY

In [Ref. 1], Crigler lists several additional topics for further study, some of which were done here. Along with those in Reference 1, additional recommended study topics are:

(1) Develop an expression for the probability that a specified number of messages are successfully received by some time, t, taking retransmissions into account.

(2) Investigate the changes in the results of the analytical and simulation models using a variable message length rather than constant.

(3) Develop a mathematical expression for the optimum number of channels for a successful system based on the number of originators.

(4) Investigate the effects of different values of the interval (0,L).

(5) Investigate the transient state of the system before the arrival of the transmission constant.

STORAGE VECTOR (X) WITH ORIGINAL TRANSMISSION START TIMES	POSITION	FINAL STORAGE VECTOR (X) WITH ALL TRANSMISSION START TIMES
6.342	1	6.34
11.848	2	11.85
34.331	3	34.33
61.123	4	61.12
81.277	5	81.28
96.327	6	96.33
148.317	7	148.32
151.251	8	151.25
153.092	9	153.09
175.865	10	175.87
187.388	11	187.39
193.900	12	193.90
198.921	13	198.92
200.715	14	200.72
202.547	15	202.55
218.205	16	218.21
219.994	17	219.99
246.827	18	240.90
259.204	19	246.83
292.429	20	247.72
	21	259.20
	22	287.90
	23	292.43
	24	294.72
	25	334.90
	26	339.43

$c = 10,\ n = 20$

TRANSMISSION MATRIX (XT) WITH ALL MESSAGES SUCCESSFULLY RECEIVED

Channel	1	2	3	4	5	6	7	8	9	10
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6.34	0.0
2	0.0	0.0	0.0	11.85	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	34.33	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	61.12	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	81.28
6	0.0	0.0	0.0	96.33	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	148.32	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	151.25	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	153.09	0.0	0.0
10	0.0	175.87	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	187.39
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	193.90	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	198.92	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	200.72	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	202.55	0.0
16	0.0	0.0	0.0	0.0	218.21	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	219.99	0.0	0.0	0.0
18	246.83	0.0	0.0	0.0	0.0	240.90	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	247.72	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	259.20	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	287.90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	292.43	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	294.72	0.0
25	334.90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	339.43	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
49	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$c = 10;\ n = 20$

```
C
C     SIMULATION MODEL OF C-CHANNELS AND N-MESSAGES (10-20)
C
      DIMENSION X(49),XT(10,49),DIFF(10,49)
C
C     INITIAL DATA
C
      D=15.0
      XL=300.0
      H=47.0
      N=20
      IC=10
      ICI=IC+1
C
C     RANDOM NUMBER GENERATOR DATA
C
      MR=12493
      IR=87345
      KR=8*MR+3
C
C     PERFORM FIFTY ITERATIONS FOR STATISTICAL PURPOSES
C
      DO 1000 IX=1,50
C
C     ZERO VECTOR AND MATRICES IN EACH ITERATION
C
      DO 10 J=1,IC
      DO 15 I=1,49
      X(I)=0.0
      XT(J,I)=0.0
      DIFF(J,I)=0.0
   15 CONTINUE
   10 CONTINUE
C
C     GENERATE N UNIFORM RANDOM NUMBERS OVER INTERVAL (0,XL)
C
      DO 100 I=1,N
      IR=IR*KR
      R=0.5+FLOAT(IR)*2.328306E-10
      X(I)=XL*R
  100 CONTINUE
C
C     ORDER N UNIFORM RANDOM NUMBERS
C
      DO 150 I=1,N
      DO 200 J=2,N
      M=J-1
      IF(X(J).GE.X(M))GO TO 200
```

```
      CCUNT=X(M)
      TEST=X(J)
      X(M)=TEST
      X(J)=CCUNT
  200 CONTINUE
  150 CCNTINUE
C
C     PRINT OUT VECTOR OF ORIGINAL MESSAGE TRANSMISSION START TIMES
C
  250 FORMAT(1H ,I3,F12.3)
      DO 300 I=1,N
  300 WRITE(6,250)I,X(I)
C
C     RANDOM CHANNEL SELECTION
C
      DO 400 I=1,49
      IF(X(I).EQ.0.0)GO TO 500
      IR=IR*KR
      R=0.5+FLOAT(IR)*2.328306E-10
      LA=IC*R+1
      IF(LA.EQ.ICI)LA=IC
      J=LA
C
C     PLACE TRANSMISSION START TIMES IN TRANSMISSION MATRIX
C
      XT(J,I)=X(I)
C
C     CCMPARE TRANSMISSION START TIME WITH PRIOR TRANSMISSION START
C     TIME ON SAME CHANNEL
C
      IF(I.EQ.1)GO TO 400
      IJ=I-1
      DO 410 KK=1,IJ
      KI=I-KK
      IF(XT(J,KI).EQ.0.0)GO TO 405
      DIFF(J,I)=XT(J,I)-XT(J,KI)
      IF(DIFF(J,I).GE.D)GO TO 400
C
C     RETRANSMISSION START TIMES FOR GARBLED MESSAGES
C
      RETRY1=XT(J,KI)+H
      RETRY2=XT(J,I)+H
      GO TO 415
  405 IF(KI.EQ.1)GO TO 400
  410 CCNTINUE
C
C
C
```

```
C     PLACE RETRANSMISSION START TIMES IN MESSAGE STORAGE
C     VECTOR IN PROPER TIME ORDER
C
  415 DO 420 II=KI,49
      IF(X(II).EQ.0.0)GO TO 416
      IF(RETRY1.LE.X(II))GO TO 430
      GO TO 420
  416 X(II)=RETRY1
      GO TO 450
  430 IF(RETRY1.EQ.X(II))GO TO 450
C
C     IF NECESSARY SHIFT ALL LATER TIMES UP ONE POSITION
C
      IK=49-II
      DO 440 LI=1,IK
      IL=50-LI
      ILL=IL-1
      X(IL)=X(ILL)
  440 CONTINUE
      X(II)=RETRY1
      GO TO 450
  420 CONTINUE
  450 DO 470 III=I,49
      IF(X(III).EQ.0.0)GO TO 455
      IF(RETRY2.LE.X(III))GO TO 460
      GO TO 470
  455 X(III)=RETRY2
      GO TO 400
  460 LK=49-III
      DO 480 LL=1,LK
      KL=50-LL
      KLL=KL-1
      X(KL)=X(KLL)
  480 CONTINUE
      X(III)=RETRY2
      GO TO 400
  470 CONTINUE
  400 CONTINUE
C
C     PRINT OUT STORAGE VECTOR OF ALL TRANSMISSION START TIMES
C
  500 DO 600 IW=1,49
      IF(X(IW).EQ.0.0)GO TO 550
 9100 FORMAT(1H ,I5,F10.2)
      WRITE(6,9100)IW,X(IW)
  600 CONTINUE
C
C
```

```
C        PRINT OUT TRANSMISSION MATRIX
C
  550 DO 700 IWW=1,49
 9200 FORMAT(3X,I3,10F7.2)
      WRITE(6,9200)IWW,(XT(J,IWW),J=1,10)
  700 CONTINUE
 1000 CONTINUE
 9999 STOP
      END
```

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13. ABSTRACT

A satellite communication system has been proposed for use in improving naval communications. This system would feature random selection of channels by originators. For a message to be received successfully it must be alone on a channel during its transmission period. The number of transmissions by each unit and the total system transmission time must be minimized to prevent detection by enemy direction finding equipment. An expression is developed for the probability that a specified number of messages, s, of n original messages, will be received successfully after being transmitted a number of times, m. An expression is derived for the probability that a specified number of messages, j, of n original messages, will be received successfully on the first transmission by some arbitrary time, t. A Monte Carlo computer simulation is conducted and is the basis for expressions for the maximum number of times any unit would have to transmit to be received successfully.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Transmission Start Time						
Message Generation						
Retransmission Delay						
Generation Interval						

Jones 131112

Models for message transmissions and interference-caused retransmission through a multi-channel satellite communications system.